Planarity of Force Tilings in Jammed Packings of Disks

Kabir Ramola

Martin Fisher School of Physics,
Brandeis University

In collaboration with Bulbul Chakraborty

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Jammed Configurations

- We study jammed systems of disks in two dimensions.

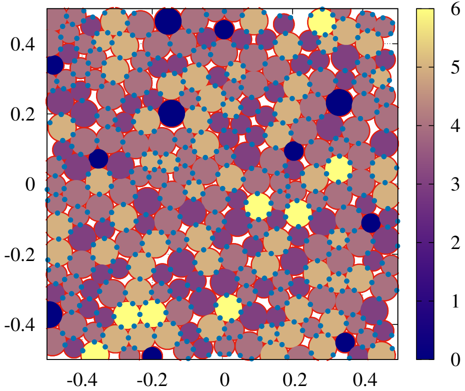


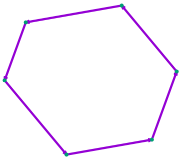
Figure: A jammed packing of 256 bidispersed frictionless disks coloured according to their contact numbers.

Force Polygons

- The configurations are in **mechanical equilibrium**.
- The forces at every grain **sum to zero**,

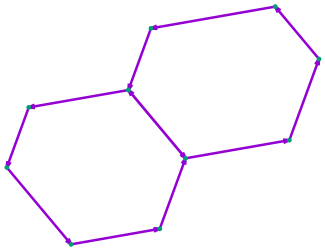
$$\sum_c \vec{f}_{g,c} = 0. \tag{1}$$

- The "vector sum" of the forces on each grain form a **closed polygon**.



- If the sum is taken **cyclically** over the contacts for each grain, we obtain **convex polygons** for frictionless systems.

Force Tiles



- Newton's **third law** dictates that

$$\vec{f}_{g,c} = -\vec{f}_{g',c}, \tag{2}$$

  at the contact c between the grains g and g'.
- We can therefore use this fact to **tile polygons** next to each other and produce a **force tile network**.

Force Tiles

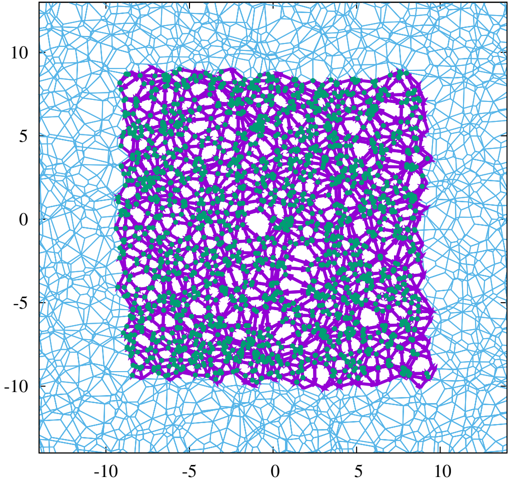


Figure: The force tiling associated with a jammed packing of 2048 disks.

Force Tiles

Height Fields

- This naturally leads to the description of the system in terms of **height variables**.

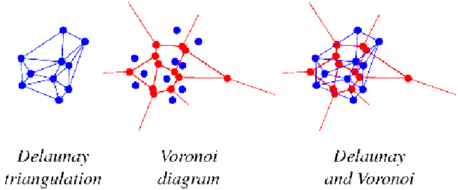
- The force at each contact is given by the **difference of height variables**

$$\vec{f}_{g,c} = \vec{h}_{g,v} - \vec{h}_{g,v'}. \tag{3}$$

- The **height vectors are the vertices of the force tiling**.

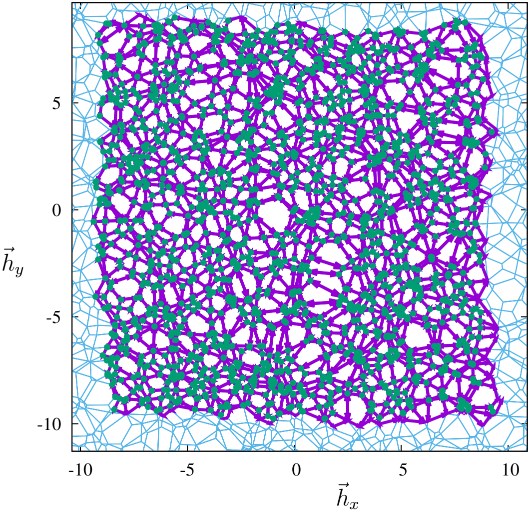
- The height vectors are **associated with the dual lattice** of the contact network i.e. the network of voids $\{v\}$.

Delaunay Triangulation

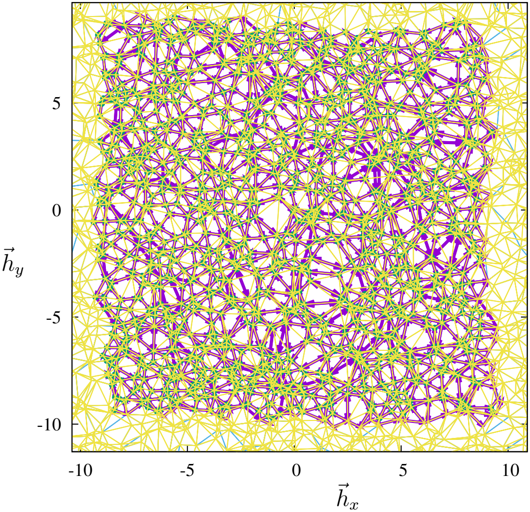


- A Delaunay triangulation is the **adjacency graph of Voronoi tesselation**.
- It **maximizes the minimum angle** of all possible triangulations.
- A **circumcircle** of any Delaunay triangle **does not contain any other points** in its interior.
- **Nearest neighbour graph is a subgraph** of the Delaunay.

Force Tiles and Delaunay Triangulation



Force Tiles and Delaunay Triangulation

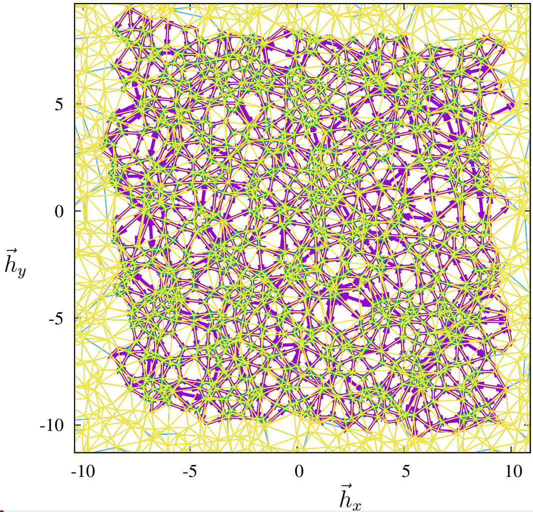


Overlap Parameter

- The force tiling represented by the set of **height vertices and edges forms a network** $\mathcal{G} = (V, E)$.
- A Delaunay triangulation of these vertices then **forms a related network** $\mathcal{G}_D = (V, E_D)$.
- We define a planarity order parameter ψ as the **overlap of the two graphs** $\psi = \langle \mathcal{G}_D | \mathcal{G} \rangle$.
- We find signatures of the existence of **non-planar and planar phases** as a function of external load.
- We study this behaviour using **simulations of frictionless** soft disks and **experimental data of frictional** disk packings.

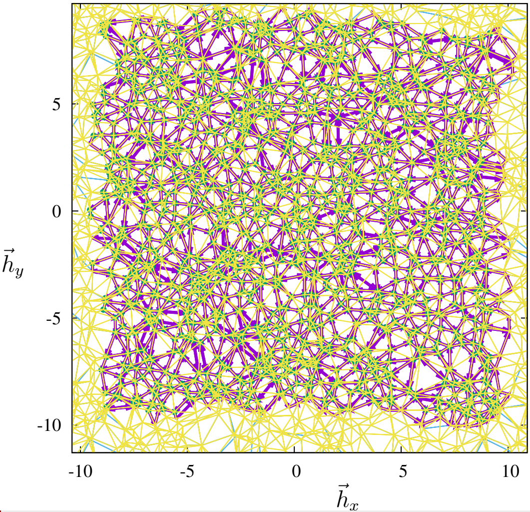
Frictionless Systems

Energy $= 10^{-13}$



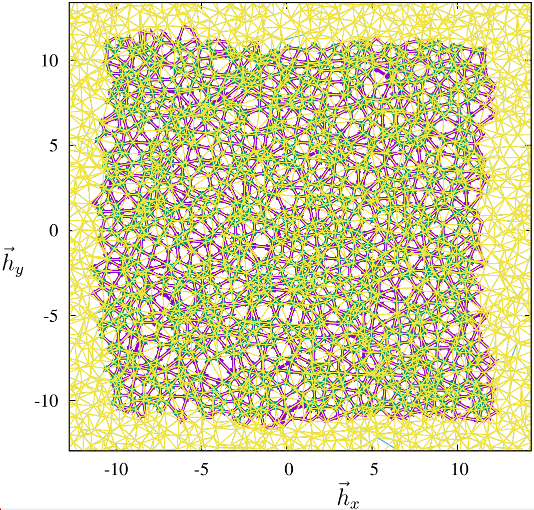
Frictionless Systems

Energy $= 10^{-7}$

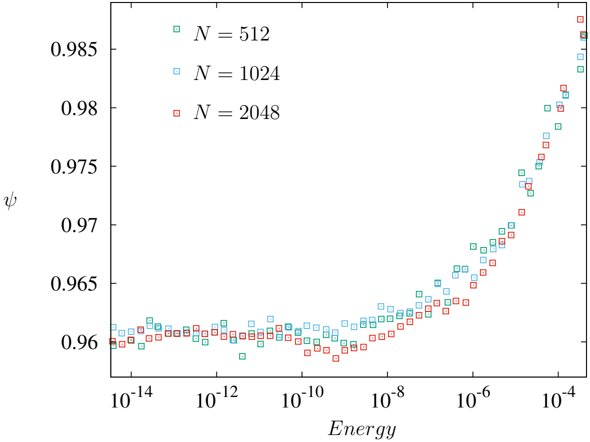


Frictionless Systems

Energy $= 10^{-3}$



Overlap Parameter: Frictionless Systems



Frictional Systems

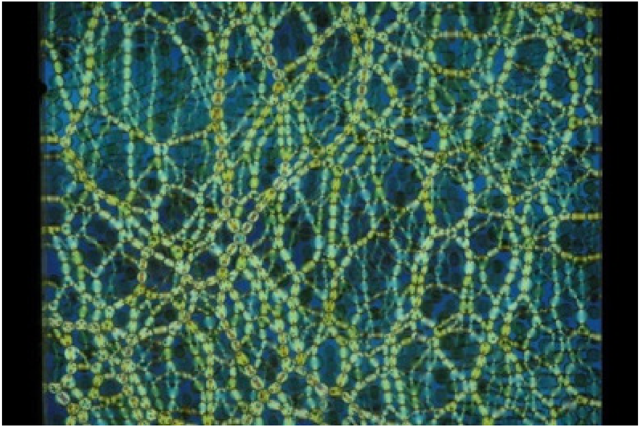
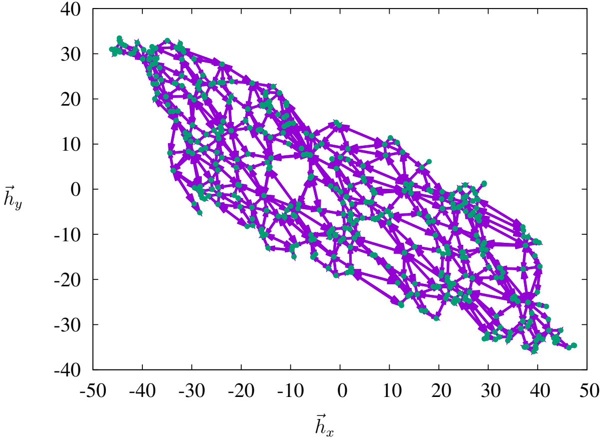


Figure: Shear Jamming experiment from Bob Behringer's group, Duke University

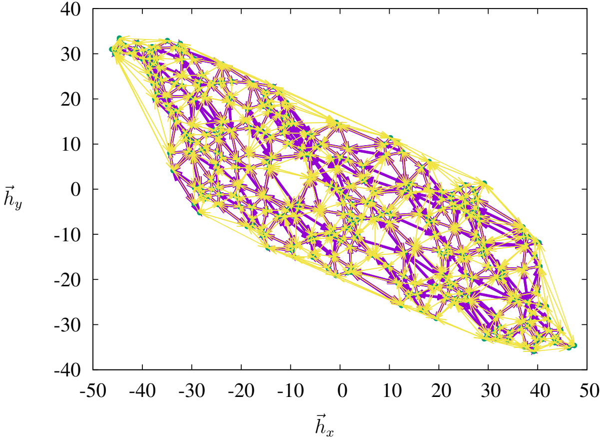
Frictional Systems

Strain Step = 20



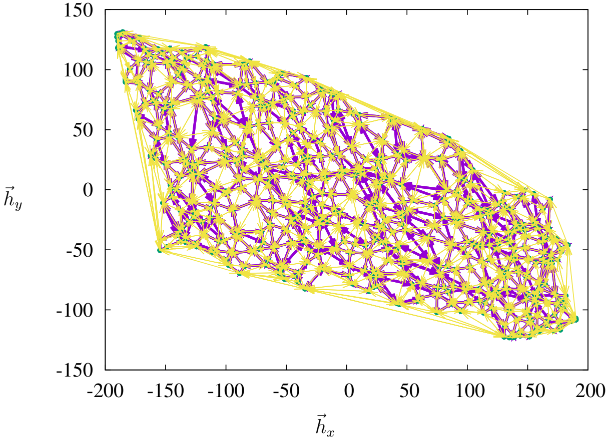
Frictional Systems

Strain Step = 20

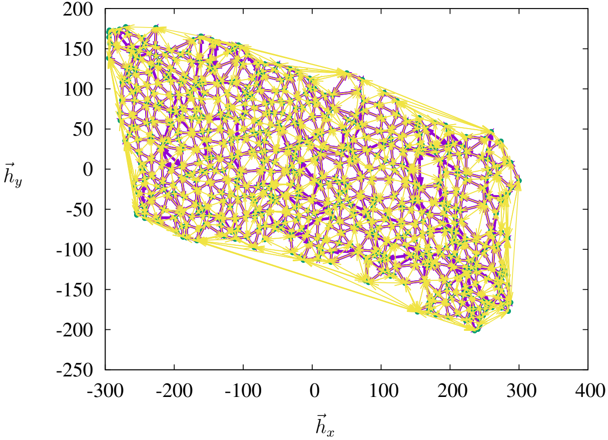


Frictional Systems

Strain Step = 40

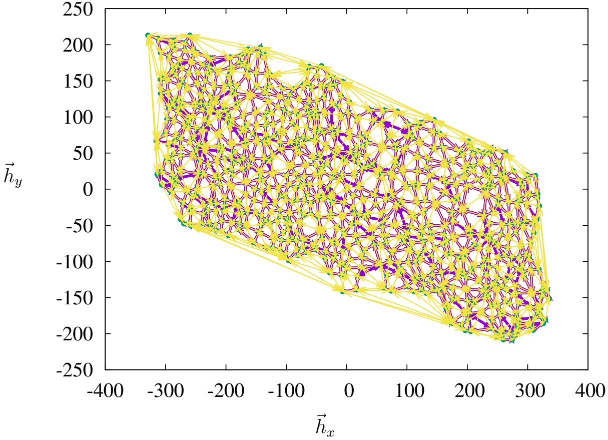


Strain Step = 60

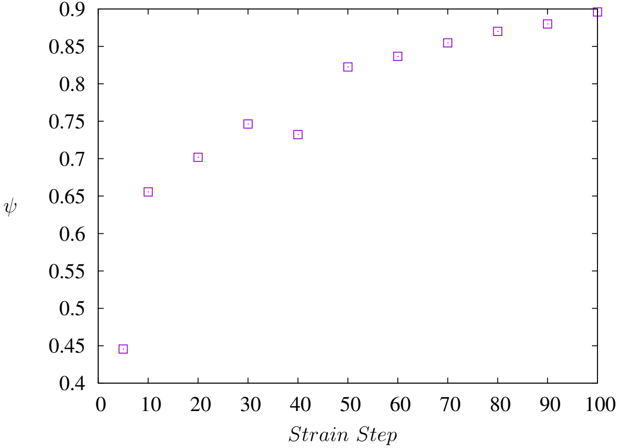


Frictional Systems

Strain Step = 80



Overlap Parameter: Frictional Systems



Thank You.